Logical Quantum Computing

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Quantum computing has the potential to revolutionize a wide array of industries, from pharmaceuticals and materials research to finance and logistics, by offering a fundamentally different way of processing information using quantum mechanical systems. The promise of quantum computing relies on the ability to store and process information in quantum bits (qubits), which are notoriously fragile. It is exceedingly difficult to simultaneously isolate a quantum system from noise and to control it precisely. The idea of quantum computing first arose in the context of quantum simulation; Richard Feynman proposed in the 1980s that to properly simulate quantum systems, one must use a quantum computer (Feynman 1982), but quantum computing was thought to be impractical due to the inability to control qubits without errors. Quantum computers are made up of individual quantum bits that are coupled to noisy environments, which unlike classical computers cannot rely on redundancy to prevent errors. Additionally, the state of a quantum bit (or qubit) can be a linear superposition of $|0\rangle$ and $|1\rangle$. This means that any error correction must preserve additional phase information of the qubit (Gottesman 2009).

The field of quantum computing was spurred on by discoveries of quantum error correction (QEC) codes in the 1990s (Shor 1995, Steane 1996, Laflamme, et al. 1996). These codes, along with others that have been since been developed, all work by encoding a logical qubit into the space of many physical qubits. (Shor 1995) describes the circuit for Shor's error correction code, also referred to as the repetition code. This scheme works by encoding a logical qubit into nine physical qubits using the following definitions of the logical states:

$$|0_L\rangle = (|000\rangle + |111\rangle) \otimes (|000\rangle + |111\rangle) \otimes (|000\rangle + |111\rangle)$$

$$|1_L\rangle = (|000\rangle - |111\rangle) \otimes (|000\rangle - |111\rangle) \otimes (|000\rangle - |111\rangle)$$

A bit flip can then be detected and corrected based on majority voting, i.e. the state $|100\rangle + |011\rangle$ with an error on the leftmost qubit is returned to $|000\rangle + |111\rangle$. Similarly phase flips are detected based on sign changes between the groupings of three qubits.

Since these codes first proved that quantum error correction is possible, many more codes have been developed. More recent codes include stabilizer codes, of which there are many variants each having different requirements for numbers of qubits and error thresholds. One typical example of a stabilizer code is the surface code (Bravyi and Kitaev 1998), a topological code defined on a 2D lattice of qubits that is currently popular for those designing hardware around a quantum error correction (QEC) code architecture. The advantage of the surface code is that it has a relatively high error threshold (the level of errors that can be corrected) and requires only nearest neighbor connectivity.

Recent experiments have demonstrated various building blocks of the surface code architecture (Corcoles, et al. 2015, Takita, et al. 2016, Riste, et al. 2015, Kelly, et al. 2015). The number of physical qubits needed to build a logical qubit depends on the type of errors and the error rates present on the physical level. In Shor's QEC code, the logical qubit consists of nine physical qubits, one data qubit and eight ancillae, and it corrects for both phase and bit flip errors on the data qubit. Errors can also occur in the ancilla qubits as well as the data qubit, and an encoding into a larger number of physical qubits is necessary to correct for those second order errors.

In the surface code framework, the smallest logical qubit that corrects for both phase (Z) and bit flip (X) errors needs 17 physical qubits. A fully fault tolerant quantum computer based on the surface code assuming realistic error rates is predicted to require millions of physical qubits. Figure 1(a) shows schematic for a single logical qubit in a variation on the surface code called the rotated surface code. This logical qubit is built out of 49 physical qubits on a 7x7 square grid. A logical bit flip gate is performed by individual bit flip gates along the logical-X boundaries (indicated in red). Likewise, phase gates along the Z-boundaries (in blue) accomplish a logical phase flip. During computation, the physical qubits are initialized as eigenstates of both X- and Z-stabilizers, operators like XXXX and ZZZZ that track bit-flip parity and phase-flip parity respectively, and are maintained in these states by X- and Z- parity checks performed by the circuits in Figure 1(b).

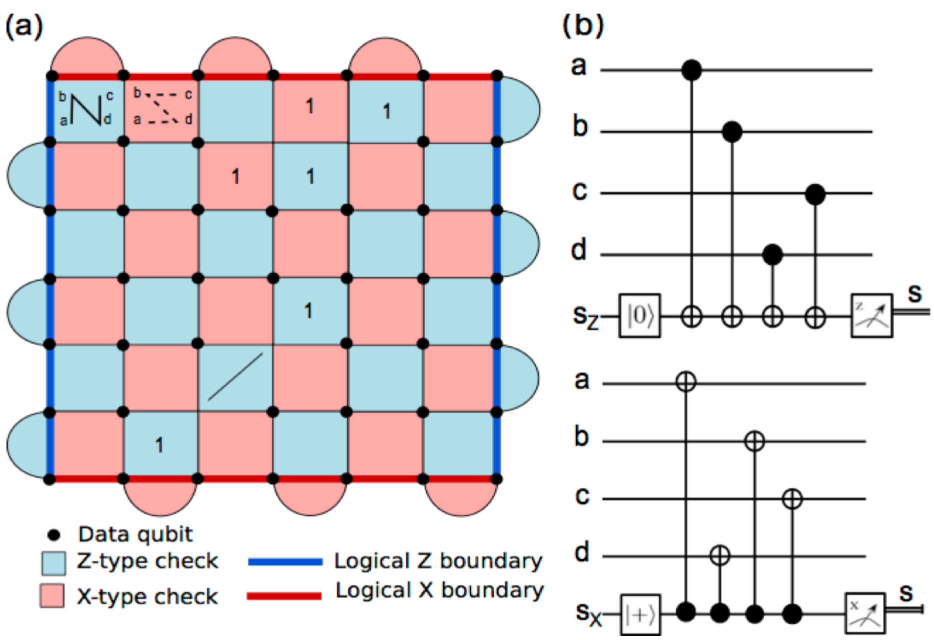

Figure 1: (a) Schematic of the rotated surface code. (b) X- and Z-parity check circuits. Figure from (Gambetta, Chow and Steffen 2017)

Hybrid Quantum Computing

A number of quantum computing devices are now available as cloud services both freely to the public and as commercial offerings, including up to 20 qubit devices on superconducting qubit platforms. While these devices mark significant advancements in the field of quantum computing and may be referred to as “quantum computers”, it is important to note that these small devices are far from the ultimate goal of fault-tolerant universal quantum computers. Existing devices are noisy and contain small enough numbers of qubits that they can still be simulated classically (although they are quickly approaching the limits of classical simulation).

Near-term devices pose an interesting problem however: once devices have enough qubits and can perform long enough depth circuits that they cannot be classically simulated but do not have error correction, can they do anything useful? While development of fault-tolerant quantum algorithms remains a vigorous area of active research, the availability of noisy intermediate-scale quantum (NISQ) devices is stimulating new efforts for finding applications that do not require fault tolerance. These applications may include quantum chemistry (Yung, et al. 2014, Kandala, Mezzacapo, et al. 2017), optimization, and machine learning.

Recent experiments have demonstrated hybrid quantum-classical algorithms such as variational quantum eigensolvers (VQE) (McClean, et al. 2016, Farhi, Goldstone and Gutmann 2014). These experiments are less sensitive to gate errors because they involve a classical optimization step. The procedure is to prepare a trial state on the quantum processor and then measure some observables (how the state is prepared and which observables are measured depends on the problem being solved); then based on those observables, the state preparation parameters are updated on a classical computer and run again until a minimum value is achieved. For example, (Kandala, Mezzacapo, et al. 2017) uses VQE to find the ground state of small molecules.

In the chemistry experiments, a fermionic Hamiltonian is mapped to qubits; (Kandala, Mezzacapo, et al. 2017) find this mapping following the procedure from (Bravyi, et al. 2017). The trial state is prepared by applying a sequence of alternating single qubit gates and entangling steps, where each single qubit gates is parameterized by three phases. Then the observables that correspond to the qubit Hamiltonian are measured, and the energy for that state is calculated. The classical computer updates the phase parameters according to a minimization algorithm, simultaneous perturbation stochastic approximation (SPSA) for this work (Spall 1992), and the cycle repeats until a minimum energy is found.

This approach is referred to as hybrid quantum computing or approximate quantum computing. In addition to having inherently looser requirements on error rates due to the optimization process, these experiments can be further improved by techniques like error mitigation (Kandala, Temme, et al. 2018). This technique works when the errors all scale with the length of the gates applied during the experiment. One can run the original experiment once and then repeat with all operations slowed down. The repeated experiment will have larger errors, but the two sets of data can be used to extrapolate to the zero-noise limit via Richardson extrapolation (Richardson 1911). While both chemistry and machine learning applications have been demonstrated experimentally with error mitigation, these experiments are all on small devices with only a few qubits. Near term applications will need to scale up these experiments to larger numbers of qubits to establish a quantum advantage.

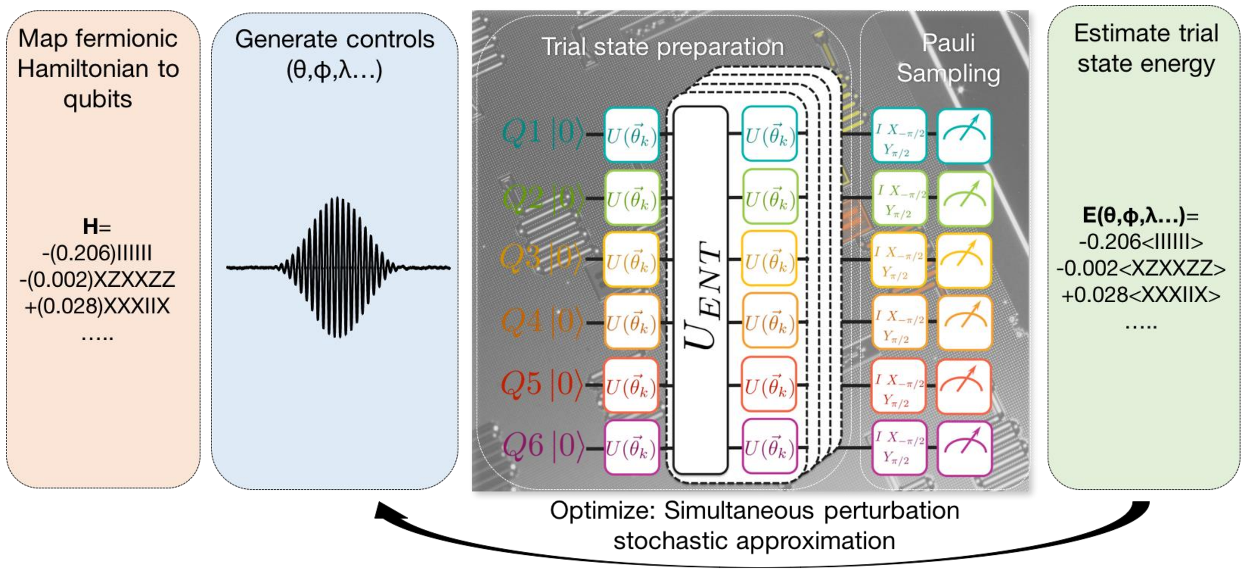

Figure 2: Steps for finding the ground state energy using VQE.

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